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Sliding mode observer-based control for a class of bioreactors

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Abstract

A robust control algorithm which uses partial state feedback is designed for a class of biochemical processes in the presence of modeling uncertainties. To design the controller, the model uncertainty and the nonmeasurable state are combined into a new state variable. A sliding mode state observer is used to obtain on-line estimates of this new state. A practical stabilizer is obtained by combining the observer with an input–output linearizing controller. The practical convergence of the observer and the controller are proved. The performance of the sliding mode observer and the closed-loop behavior is illustrated through numerical simulations.

Keywords: Bioreactor; Sliding mode; Simulations; Scale observers

1. Introduction

During the last decade, there has been a growing interest on biotechnological processes. These processes requires high-performance control techniques due to increased demands on productivity, product quality and environmental responsibility. However, two of the major obstacles in the application of computer control algorithms for biotechnological processes are the difficulty of modeling the growth kinetics of microorganisms and the lack of reliable, sterilizable and robust sensors for the on-line measurements of process key variables such as biomass, substrate, precursor, and product concentrations [20]. Sensors in the biotechnological field are generally less reliable than their counterparts in other industries [11]. The non-measurable variables in a bioreactor are obtained using indirect techniques [19,21,22]. The first attempts to estimate unknown variables related to continuous bioreactors involved macroscopic component balances. Some of these approaches involved batchwise analyses which are done manually. Therefore, these techniques are time consuming, require a lot of manpower and may result in very expensive solutions as far as the measurements of very specific compounds are concerned. One way to avoid these problems is to use estimation strategies. An excellent review of estimation techniques is provided in [4]. Nowadays, there is a growing interest in the development of estimation strategies to control continuous bioprocesses, which are highly nonlinear processes. Estimation and control strategies based directly on nonlinear models may improve performance [16]. Many of the strategies to estimate nonmeasurable states and disturbances for partially known systems are based on the extended Kalman filter (EKF) and variations thereon [9,11,12,16]. Some other works are based on high-gain observers [4,17,18,24].

For nonlinear processes whose nonlinearities are strong, linear controller design techniques often become inadequate when sufficient large excursions from steady state takes place. For these processes, more efficient alternatives must be considered. A great development in nonlinear control theory in the last few years has been the characterization of linearizable systems, i.e. systems that can be linearized by means of a change of coordinates and state feedback. Linearization of nonlinear systems is related with the cancellation of input/output nonlinearities under the assumption of a perfect knowledge of these terms. A feedback control scheme designed with this approach guarantees closed-loop stability and output tracking [13]. In general, the problem of obtaining an exact knowledge of the nonlinearities present in a system is not an easy task. Since the nonlinear terms are not fully known, the linearizing controller can provide a poor performance or even induce instabilities [10]. For the stabilization of chemical reactions there are several approaches which are based on Taylor linearization of the reactor

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dynamics. These works assume that the uncertainties belong to certain conic sector [3,15]. These approaches have several weaknesses. The main properties of the chemical reactors cannot be exploited when the model is locally linearized. In addition, many uncertainties and disturbances cannot be included in a conic sector. This situation can lead to conservative control law designs and consequently poor closed-loop performance.

Recently, new techniques to get on-line estimates of the uncertainty terms in chemical reactors, for both modeling and control purposes, have been developed. These techniques use filtering [9] and calorimetric balances [22]. The advantage of these approaches is their easy computational implementation. Besides, their structure has a strong physical meaning, without the restrictions of the methods mentioned above. However, the calorimetric balance techniques become unstable when the measurements are noisy.

In the spirit of calorimetric balances, another kind of observer structures can be used: sliding mode observers (SMO). SMO are robust observers which estimate the state of a nonlinear uncertain system. SMO are well suited for nonlinear uncertain systems with partial state feedback [17]. The main advantage of SMO, over a linear observer such as a Luenberger observer, is that SMO can be made considerably more robust to parametric uncertainty, external disturbances and noisy measurements [27]. The main drawback of SMO is that extensive efforts in the design procedure must be taken to guarantee robustness for bounded modeling errors [25]. The aim of this work is to show that SMO can be used successfully to design a robust control strategy for a class of bioreactors. The central idea of this approach is to consider the modeling uncertainties as a new non measurable state variable. The resulting controller is a practical one and only substrate concentration measurements are required for its implementation. We provide the conditions for the convergence of the observer and the controller. Some of the observer parameters have physical meaning.

This work is organized as follows. In the next section, the class of bioprocesses to be studied is discussed and a precise statement of the problem is presented. To cope with the lack of on-line biomass concentration measurements and the presence of model uncertainties, an SMO is proposed in Section 3. In addition, using this observer, an observer-based input–output linearizing controller is obtained. The conditions for the convergence of the observer and the controller are established in this section. Section 4 illustrates the open-loop and closed-loop dynamic behavior through numerical simulations. Section 5 describes the future work to eliminate the chattering effect. Section 6 concludes the work.

2. System description

Bioreactors are generally regarded as containers which are used to synthesize products by means of biochemical reactions. In a bioreactor, microorganisms utilize available nutrients for growth, biomass maintenance, and product formation. The most important bioreactor for industrial application is the conventional mixing vessel which has the advantages of low cost and low operating costs.

In this work, the process is composed for a simple microbial culture, which involves a single biomass growing on a single substrate and yielding a single product. The reactions take place in a continuous stirred tank reactor (CSTR) and there are no cells in the feed. This system was chosen because, despite this process being the simplest one, its dynamical behavior is complex [1]. Moreover, several important industrial processes belong to this class (e.g. wastewater treatment process). In the next subsection, the mathematical model to describe the bioreactor is presented.

2.1. Mathematical model

The function of a biological model is to describe the metabolic reaction rates and their stoichiometry on the basis of present and past bioreactor conditions. The models used to describe bioreactors can be divided into unstructured and structured ones [5].

The unstructured models are the simplest. The unstructured models are based on the assumption that biomass and substrate can be adequately described by single parameters. In many cases this assumption is reasonable because it is impossible to have exact knowledge of the heterogeneous composition of the biomass and the state of the intracellular systems. Unstructured models describe a condition called balanced growth. This is a quasi-steady state assumption, which requires that the environment of the biomass changes sufficiently slowly so that the biomass can adjust its internal composition to adapt to the changes.

In unstructured models the microorganisms (biomass) and nutrients (substrate) are viewed as homogeneous components. In a process with a single biomass growing on a single substrate, the variables that define the system are biomass z (g/l) and substrate s (g/l) concentrations in the bioreactor. Growth of microbial cells in a suitable medium results in the consumption of substrate and sometimes the formation of products. The dynamic behavior of z and s are obtained using mass balances for a CSTR. The mass balances lead to the next system of ordinary differential equations

$$\dot{z} = -Dz + \mu(s)z \tag{1}$$

$$\dot{s} = -D(s_f - s) - \frac{\mu(s)z}{Y_d(s)} \tag{2}$$

where D is the dilution rate (h^{-1}), s_f the inlet substrate concentration (g/l), $Y_d(s)$ the yield production coefficient and $\mu(s)$ is the specific growth rate (h^{-1}). The second term in (1) represents the microorganisms' growth rate. Correspondingly, the second term in (2) represents the substrate consumption rate.

Several basic types for the specific growth rate ($\mu(s)$) have been proposed [5]. Despite some objections to the Monod's equation, this one and variations thereon are probably the most widely used, primarily because they are simple and mathematically tractable. The Monod's equation is analogous to the Michaelis–Menten enzyme kinetics and can be expressed as

$$\mu(s) = \frac{\mu_m s}{K_m + s} \tag{3}$$

where μ_m and K_m are positive constants which are characteristics of the organisms and growth substrate.

In an experimental continuous bioreactor, Dibiasio et al. [7] verify the existence of multiple steady states. Through steady-state analysis, several researches [1,8] conclude that nontrivial multiplicity and oscillatory phenomena were not possible for a constant yield production coefficient. The yield production coefficient (Y_d) is an extremely important parameter since it represents the efficiency of conversion of the substrate into biomass. Agrawal et al. [1] found that in a bioreactor with specific growth rate given by Monod's equation, a linearly increasing yield coefficient

$$Y_d(s) = a' + b's \tag{4}$$

leads to a dynamic behavior characterized by a self-sustained oscillation, where a' and b' are positive constants.

2.2. Main assumptions

The main theoretical assumptions for the development of the control strategy are listed below. These assumptions are made using some physical properties of the systems under study.

Assumption 1 (A1). Both concentrations (substrate and biomass) have finite values, which can be mathematically expressed as

$$0 \le s(t) \le s_f, \qquad 0 \le z \le z_{\max}$$

Remark 1 (R1). *This assumption is in accordance with the experimental evidence. The continuous bioreactors display a self-regulatory property. During growth, new cell mass is formed autocatalytically from substrate. Microbial reactions usually show saturation at high substrate concentrations, i.e. reaction rates approaches a maximum value. On the other hand, the reaction rates equal zero if no substrate is available.*

Assumption 2 (A2). On-line measurements of the substrate concentration are available.

Assumption 3 (A3). On-line measurements of the biomass concentration are not available.

Remark 2 (R2). *For most of the bioprocesses,* A2 *and* A3 *are realistic assumptions. In general, getting estimates of the biomass concentration is an expensive procedure and it induces very large delays on the closed-loop system.*

Assumption 4 (A4). The functions for the specific growth rate and the yield production coefficient are bounded and unknown.

Remark 3 (R3). *In general, the kinetic rates involved in biological processes are very complex functions of the operating conditions and of the state of the process. Therefore, in accordance with* A3 *its value cannot be obtained. In addition, the analytical modeling of these functions is often cumbersome and still constitutes the subject of continuing and intensive investigations.*

2.3. Problem statement

The control objective for the bioreactor given by (1) and (2) is to regulate the substrate concentration (control output) at a prescribed set point (s^{ref}) manipulating the dilution rate (control input).

Let us define the regulation error as: $\bar{s}(t) = s(t) - s^{\text{ref}}$. Suppose that the closed-loop performance for the regulation error is specified through the desired closed-loop time constant $\tau_c > 0$. This means that the desired closed-loop performance is

$$\dot{\bar{s}} + \tau_c^{-1}\bar{s} = 0 \tag{5}$$

which guarantees that

$$\lim_{t\to\infty} \bar{s}(t) = 0$$

Following [13], the linearizing *ideal* control law for the substrate concentration (2) can be computed as

$$D = \frac{1}{s_f - s}\left(\tau_c^{-1}\bar{s} - \frac{\mu(s)z}{Y_d(s)}\right) \tag{6}$$

The bioreactor has a relative degree of unity [13]. Then, the biomass concentration defines the internal dynamics for the bioreactor. In accordance with A1, the internal dynamics is stable. It is clear that the ideal control law (6) cannot be applied just as it is because, in accordance with A3 and A4 the term $\mu(s)z/Y_d(s)$ is unknown.

The problem to face in this work is to control a partially known system. To control this kind of processes, several techniques have been proposed [4,6,17,24].

To overcome the lack of on-line values of the substrate consumption rate $\mu(s)z/Y_d(s)$ to compute the linearizing control law, we propose:

1. to consider the state vector composed by the substrate concentration and the substrate consumption rate ($\mu(s)z/Y_d(s)$);
2. to use an SMO to get on-line estimates of the substrate consumption rate.

The first part of this approach has been used before [2,9,11,16,17,27]. The SMO was chosen to estimate the substrate consumption rate since the major potential advantage of the SMO over a linear observer such as a Luenberger observer, is that the sliding observer can be made considerably more robust to parametric uncertainty and external disturbances [27].

Let us define the uncertain term as

$$\psi = -\frac{\mu(s)z}{Y_d(s)} \tag{7}$$

Then, the new state vector is defined as follows:

$$\mathbf{x} = \begin{bmatrix} s \\ \psi \end{bmatrix}$$

The dynamics of the uncertain term is given by

$$\dot{\psi} = \gamma(s, \psi) = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu(s)z}{Y_d(s)}\right) \tag{8}$$

Since $\mu(s), x$ and $Y_d(s)$ are not known, $\gamma(s, \psi)$ is also unknown. To get a reliable control law, in the next section an SMO is used to obtain an estimate ($\hat{\psi}$) of the uncertain term.

3. Estimator and controller design

SMO is a high performance state estimator well suited for nonlinear uncertain systems with partial state feedback [17]. The sliding function of this observer is the estimation error of the available output. The basic SMO structure consists of switching terms added to a conventional Luenberger observer. In Section 3.1 a brief review of SMO is presented [22].

3.1. Sliding observer for SISO systems

Consider the state space representation of a second order nonlinear system in noncompanion form

$$\dot{x}_1 = f_1(x_1, x_2), \qquad \dot{x}_2 = f_2(x_1, x_2) \tag{9}$$

where $\mathbf{x} = [x_1, x_2]^{\mathrm{T}}$ is the state space vector and x_1 is assumed to be the only measurable state. $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are nonlinear unknown functions.

The observer task is to estimate the state **x**. The observer dynamics can be written as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{f}_1(\hat{x}_1, \hat{x}_2) - \alpha_1\tilde{x}_1 - k_1\,\mathrm{sgn}(\tilde{x}_1) \\ \dot{\hat{x}}_2 &= \hat{f}_2(\hat{x}_1, \hat{x}_2) - \alpha_2\tilde{x}_1 - k_2\,\mathrm{sgn}(\tilde{x}_1) \end{aligned} \tag{10}$$

where α_1, α_2, k_1 and k_2 are positive numbers. $\tilde{x}_1 = \hat{x}_1 - x_1$ is the estimation error. $\hat{f}_1(\hat{x}_1, \hat{x}_2)$ and $\hat{f}_2(\hat{x}_1, \hat{x}_2)$ are estimated values of $f_1(\hat{x}_1, \hat{x}_2)$ and $f_2(\hat{x}_1, \hat{x}_2)$, respectively. The function $\mathrm{sgn}(\cdot)$ is defined as follows:

$$\mathrm{sgn}(\tilde{x}_1) = \begin{cases} -1 & \text{if } \tilde{x}_1 < 0 \\ 0 & \text{if } \tilde{x}_1 = 0 \\ 1 & \text{if } \tilde{x}_1 > 0 \end{cases}$$

Using (9) and (10) the resulting error dynamics can be written as

$$\begin{aligned} \dot{\tilde{x}}_1 &= \Delta f_1 - \alpha_1\tilde{x}_1 - k_1\,\mathrm{sgn}(\tilde{x}_1) \\ \dot{\tilde{x}}_2 &= \Delta f_2 - \alpha_2\tilde{x}_1 - k_2\,\mathrm{sgn}(\tilde{x}_1) \end{aligned} \tag{11}$$

where $\Delta f_1 = \hat{f}_1(\hat{x}_1, \hat{x}_2) - f_1(\hat{x}_1, \hat{x}_2)$ and $\Delta f_2 = \hat{f}_2(\hat{x}_1, \hat{x}_2) - f_2(\hat{x}_1, \hat{x}_2)$.

The sliding condition for (11) can be expressed as

$$\tilde{x}_1\left(\Delta f_1(\hat{x}_1, \hat{x}_2) - \alpha_1\tilde{x}_1 - k_1\,\mathrm{sgn}(\tilde{x}_1)\right) < 0 \tag{12}$$

Once the sliding takes place (i.e. $\tilde{x}_1 = 0$) the resulting error dynamics takes the form

$$\dot{\tilde{x}}_2 = -\frac{k_1}{k_2}\Delta f_1 + \Delta f_2 \tag{13}$$

Some important issues regarding the above observer depends on the structure of Δf_1 and Δf_2. System (9) is observable if f_1 is a single valued function of x_2. In addition, Δf_1 must be a function of $\tilde{x}_2$ in order for the control term $-(k_1/k_2)\Delta f_1$ to have an influence on the error dynamics.

Remark 4 (R4). *The sliding mode of system* (9) *can be reached if k_1 and k_2 are different from zero.*

3.2. Application to a class of bioreactors

In this section, the SMO previously described is applied to the bioreactor given by (1) and (8), which can be rewritten under the form (9) as follows:

$$\dot{s} = f_1(s, \psi), \qquad \dot{\psi} = f_2(s, \psi) \tag{14}$$

where $f_1(s, \psi) = -D(s_f - s) + \psi$ and $f_2(s, \psi) = \gamma(s, \psi)$. Note that $f_1(s, \psi)$ in (14) is a single valued function of ψ, thus, system (14) is observable.

The SMO (10) for system (14) is

$$\begin{aligned} \dot{\hat{s}} &= \hat{f}_1(s, \hat{\psi}) - \alpha_1\tilde{s} - k_1\,\mathrm{sgn}(\tilde{s}) \\ \dot{\hat{\psi}} &= \hat{f}_2(s, \hat{\psi}) - \alpha_2\tilde{s} - k_2\,\mathrm{sgn}(\tilde{s}) \end{aligned}$$

where $\tilde{s} = \hat{s} - s$. In this work, the estimated values for $f_1(s, \psi)$ and $f_2(s, \psi)$ are chosen as follows:

$$\hat{f}_1(s, \hat{\psi}) = -D(s_f - s) + \hat{\psi} \qquad \hat{f}_2(s, \hat{\psi}) = \Gamma$$

Note that $\hat{f}_1(s, \hat{\psi})$ involves the actual substrate concentration, however, since this state is measurable, the computation of $\hat{f}_1(s, \hat{\psi})$ is reliable. This modification does not change the convergence of the proposed observer. The choice of Γ will be discussed later. Note that Δf_1 is a function of $\tilde{x}_2$, which implies that the control term $-(k_1/k_2)\tilde{x}_2$ has influence on the error dynamics.

The SMO used to get an estimate of the uncertain term is

$$\begin{aligned} \dot{\hat{s}} &= -D(s_f - s) + \hat{\psi} - \alpha_1\tilde{s} - k_1\,\mathrm{sgn}(\tilde{s}) \\ \dot{\hat{\psi}} &= \Gamma - \alpha_2\tilde{s} - k_2\,\mathrm{sgn}(\tilde{s}) \end{aligned} \tag{15}$$

Using (14) and (15), the resulting error dynamics takes the form

$$\dot{\tilde{s}} = \tilde{\psi} - \alpha_1 \tilde{s} - k_1 \operatorname{sgn}(\tilde{s}) \qquad \dot{\tilde{\psi}} = \Omega - \alpha_2 \tilde{s} - k_2 \operatorname{sgn}(\tilde{s}) \tag{16}$$

where $\Omega = \Gamma - \gamma(s, \psi)$. The conditions for the existence of the sliding mode ($\tilde{s} = 0$) are

$$\tilde{\psi} \leq \alpha_1 \tilde{s} + k_1 \text{ if } \tilde{s} > 0 \quad \tilde{\psi} \geq \alpha_1 \tilde{s} - k_1 \text{ if } \tilde{s} < 0 \tag{17}$$

The sliding dynamics ($\tilde{s} = 0$) is as follows

$$\dot{\tilde{\psi}} = -\frac{k_2}{k_1}\tilde{\psi} + \Omega \tag{18}$$

Note that (18) is a filter between Ω and $\tilde{\psi}$ with a cut-off frequency at k_2/k_1. The next result shows the conditions for the practical observability of the system (12).

Proposition 1. *Suppose that Assumptions* (A1)–(A4) *hold, and choose*

$$k_2 > \xi \geq |\Omega(t)|$$

Then the system (15) *is a practical observer of system* (14), *i.e. there exists a compact set* $\mathcal{C}_1 \subset \mathbb{R}^2$ *such that all trajectories* $\tilde{\mathbf{x}}(t) = \hat{\mathbf{x}}(t) - \mathbf{x}(\mathbf{t})$ *beginning in* $\mathcal{C}_1$ *can be driven arbitrarily close to the origin.*

Proof. The integration of the sliding dynamics (18) produces the following expression

$$\tilde{\psi}(t) = \int_{t_0}^{t} \exp\left(\frac{k_2}{k_1}(\sigma - t)\right) \Omega(\sigma)\, \mathrm{d}\sigma + \tilde{\psi}(0) \exp\left(-\frac{k_2}{k_1}t\right)$$

where $\tilde{\psi}(0)$ is the initial condition for $\tilde{\psi}(t)$. Taking the absolute value of the above equation

$$|\tilde{\psi}(t)| \leq \int_{t_0}^{t} \exp\left(\frac{k_2}{k_1}(\sigma - t)\right) |\Omega(\sigma)|\, \mathrm{d}\sigma + |\tilde{\psi}(0)| \exp\left(-\frac{k_2}{k_1}t\right), \quad t > t_0 \tag{19}$$

Suppose there exists a positive constant ξ such that

$$\xi \geq |\Omega(\sigma)|$$

Eq. (19) can be expressed as

$$|\tilde{\psi}(t)| \leq \frac{k_1}{k_2}\xi \left(1 - \exp\left(\frac{k_2}{k_1}(t_0 - t)\right)\right) + |\tilde{\psi}(0)|\, \mathrm{e}^{-\gamma t} \tag{20}$$

From the above equation it is clear that

$$\lim_{t\to\infty} |\tilde{\psi}(t)| \leq \frac{k_1}{k_2}\xi$$

Recall that k_1 and k_2 are adjustable parameters. If k_2 is chosen in such a way that $k_2 > \xi$ then

$$\lim_{t\to\infty} |\tilde{\psi}(t)| \leq k_1 \tag{21}$$

The above equation means that $\tilde{\psi}(t)$ converges asymptotically to the ball $\mathfrak{B}_1(r)$ with radius $r \sim k_1$ □

Remark 5 (R5). *In accordance with Proposition* 1, k_1 *can be viewed as the desired precision in* $\tilde{\psi}(t)$.

3.3. Linearizing control law

The following step is to study the stability of the resulting closed-loop system. Note that in the dynamics of the estimation error for the uncertain term $\psi(t)$ the control input does not appear. This means that the dynamic behavior of $\psi(t)$ is independent of the control input. Then, by Proposition 1, the closed-loop dynamic behavior $\tilde{\psi}(t)$ is stable, i.e. $|\tilde{\psi}(t)| \to k_1$ as $t \to \infty$. However, the closed-loop stability of $s(t)$ must be proved.

Using the uncertainty estimation algorithm given by (15), the input–output linearizing controller for the bioreactor (14) is

$$D = \frac{1}{s_f - s}(\tau_c^{-1}\bar{s} + \hat{\psi}) \tag{22}$$

With the previous control law, the closed-loop dynamics for the substrate concentration is as follows

$$\dot{s} = -\tau_c^{-1}\bar{s} - \tilde{\psi} \tag{23}$$

Without loss of generality, let us consider the regulation control problem, that is, $\dot{s}^{\mathrm{ref}} = 0$. The dynamics for the regulation error is

$$\dot{\bar{s}} = -\tau_c^{-1}\bar{s} - \tilde{\psi} \tag{24}$$

The next result shows the conditions for practical stability of the closed-loop system.

Proposition 2. *Suppose that Assumptions* (A1)–(A4) *hold, and* k_2 *is chosen in accordance with Proposition* 1. *Then the dynamic compensator* (15), (22) *is a practical stabilizer for system* (1), *i.e. there exists a compact set* $\mathcal{C}_2 \subset \mathbb{R}$ *such that all trajectories* $\bar{s}(t) = s(t) - s^{\mathrm{ref}}$ *beginning in* $\mathcal{C}_2$ *can be driven arbitrarily close to the origin.*

Proof. Solving the tracking error dynamics (24) renders:

$$\bar{s}(t) = \int_{t_0}^{t} \exp(\tau_c^{-1}(\sigma - t))\tilde{\psi}(\sigma)\, \mathrm{d}\sigma + \bar{s}(0)\exp(\tau_c^{-1}t)$$

where $\bar{s}(0)$ is the initial condition for $\bar{s}(t)$. Taking absolute value on both sides of the last equation allows us to obtain the following inequality:

$$|\bar{s}(t)| \leq \int_{t_0}^{t} \exp(\tau_c^{-1}(\sigma - t))|\tilde{\psi}(\sigma)|\, \mathrm{d}\sigma + |\bar{s}(0)|\exp(\tau_c^{-1}t) \tag{25}$$

In the limit, when $t \to \infty$, the following expression is obtained:

$$\lim_{t\to\infty} |\bar{s}(t)| \leq \lim_{t\to\infty} \int_{t_0}^{t} \exp(\tau_c^{-1}(\sigma - t))|\tilde{\psi}(\sigma)|\, \mathrm{d}\sigma \tag{26}$$

Let $\tilde{\psi}_m = \max_t\{|\tilde{\psi}(t)| : \ t_0 \le t \le t_m\}$ be the maximum value of $|\tilde{\psi}(t)|$. This maximum value exists, by (3),(4) and (17), and Proposition 1. Then

$$\lim_{t\to\infty}\int_{t_0}^{t} \exp(\tau_c^{-1}(\sigma - t))|\tilde{\psi}(\sigma)|\,d\sigma \le \tilde{\psi}_m \lim_{t\to\infty}\int_{t_0}^{t} \exp(\tau_c^{-1}(\sigma - t))\,d\sigma \quad (27)$$

But we have

$$\tilde{\psi}_m \lim_{t\to\infty}\int_{t_0}^{t} \exp(\tau_c^{-1}(\sigma - t))\,d\sigma \le \tilde{\psi}_m \tau_c \quad (28)$$

In consequence

$$\lim_{t\to\infty} |\bar{s}(t)| \le \tau_c \tilde{\psi}_m \quad (29)$$

The last inequality implies that one can make the estimation error as small as desired, if a small enough value for τ_c is chosen, i.e. $\bar{s}(t)$ converges asymptotically to the ball $\mathfrak{B}_1(r)$ with radius $r \sim \tau_c\tilde{\psi}_m$. □

Remark 6 (R6). *The size of the ball $\mathfrak{B}_1(r)$ with radius $r \sim \tau_c\tilde{\psi}_m$, can be made as smaller as desired by choosing τ_c small enough.*

4. Numerical example

In order to illustrate the performance of the observer and the closed-loop system, numerical simulations were carried out by considering the following values for the parameters involved in the bioreactor model described in Section 2.

Parameter	Value
μ_m	0.3
K_m	1.75
a'	0.01
b'	0.03
s_f	35.0

Using these values, [25] found that the system presents two steady states, and a limit cycle is displayed around the nonwash-out steady-state ($z_s = 1.872$ g/l, $s_s = 1.531$ g/l) for a nominal dilution rate equal to $D = 0.14\,h^{-1}$. It was desired to control the system at the nonwash-out steady-state, i.e. the reference value for the substrate concentration was chosen as $s^{ref} = 1.531$ g/l. The values for the SMO parameters α_1 and α_2 were chosen using Luenberger-like observer tuning rules.

The initial conditions for the concentrations in the bioreactor were: $s(0) = 0.0$ g/l and $z(0) = 1.0$ g/l. With these conditions, the initial value of the uncertain term is $\psi = -\mu(s)z/Y_d(s) = 0$. Then, the initial conditions for the SMO were taken as: $\hat{s}(0) = 0.0$ and $\hat{\psi}(0) = 0.0$.

In all the simulations $\Gamma = 0$ because there is no prior information about the uncertain change rate ($\gamma(s, \psi)$). Then,

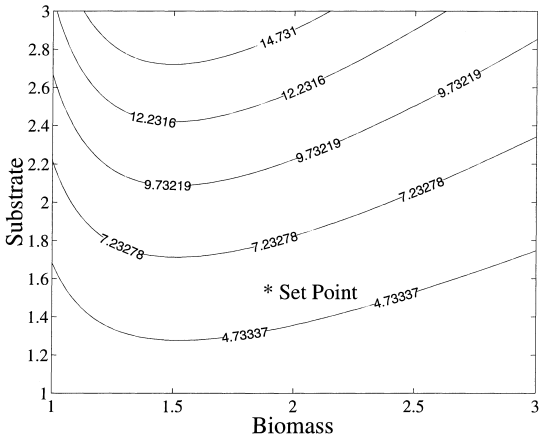

Fig. 1. Contour plot for γ around the set point.

in accordance with Propositions 1 and 2, both the observer and the controller will be practically convergent if $k_2 > |\gamma(z, s)|$. To set the value of k_2, off-line experiments can be done. Fig. 1 shows the uncertainty change rate (γ) as a function of the biomass and substrate concentrations in a neighborhood of the reference point. It is clear that around the reference point, the variations of γ are not severe. On the other hand, when all the operating conditions that the bioreactor can reach are considered, the variations of γ are severe. These variations can be explained taking into account the autocatalytic nature for most of the microbial reactions. When the biomass concentration is large, the substrate consumption rate reaches its maximum value. In spite of this, the area of operation conditions, where $\gamma(z, s)$ is almost constant is big enough.

Fig. 2 shows the effect of k_2 on the convergence of the open-loop observer. It shows two simulations with different

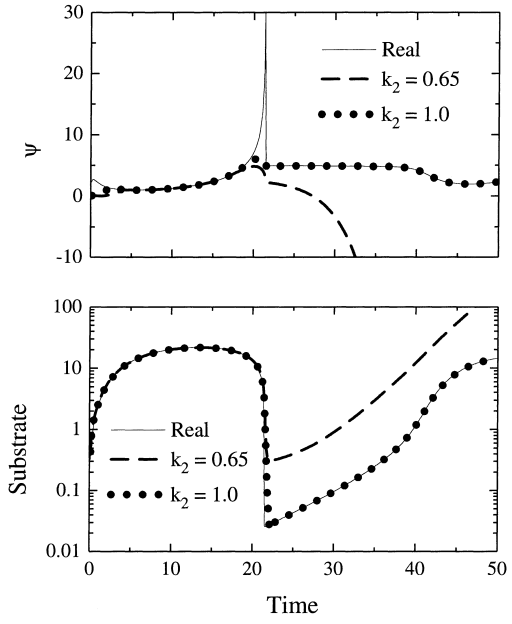

Fig. 2. Open-loop dynamic behavior for different values of k_2.

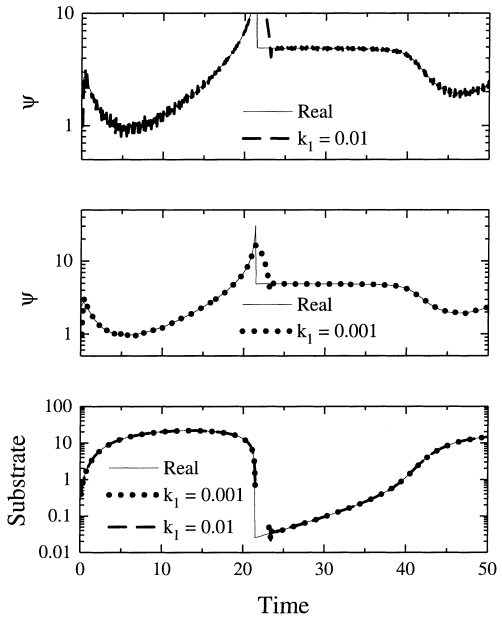

Fig. 3. Open-loop dynamic behavior for different values of k_1.

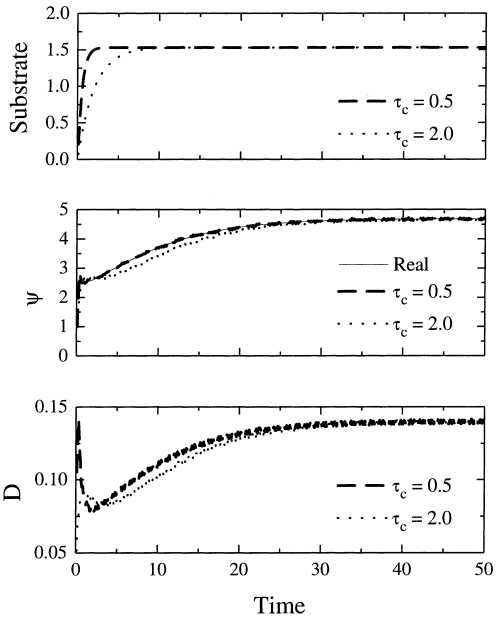

Fig. 4. Closed-loop behavior as a function of τ_c.

values of k_2 ($k_2 = 0.65$ and $k_2 = 1.0$) with $k_1 = 0.001$. This figure shows that the observer does not converge with $k_2 = 0.65$. On the other hand, with $k_2 = 1.0$, the observer displays practical convergence. It is important to notice that the largest deviations between the real and the estimated uncertainty arise at the maximum uncertainty change rate ($s \approx 0.0$). In all the simulations shown below $k_2 = 15.0$.

The effect of k_1 on the performance of the open-loop observer is analyzed in Fig. 3. Recall that k_1 is chosen as a bound on the steady state estimation error on $\tilde{\psi}(t)$. As one can see from Figs. 3a and b, reducing k_1 decreases the bandwidth for the substrate estimation error. However, reducing k_1 increases the bandwidth on the sliding patch [23].

Fig. 4 shows the closed-loop system behavior with noise-free substrate measurements using a sustained perturbation on the concentration of substrate flow feed. The sustained perturbation was taken according to the following expression:

$$s_f = s_f^0 + A\sin(\omega t),$$

where $s_f^0 = 35.0\,\mathrm{g/l}$, $A = 5.0\,\mathrm{g/l}$, and $\omega = 0.2\,\mathrm{h}^{-1}$. In this figure, two different values were taken for the closed-loop characteristic time (τ_c). The performance in both cases is satisfactory.

The major advantage of sliding mode observers is that they can be made considerably more robust to parametric uncertainties and noisy measurements. Fig. 5 shows the closed-loop time evolution of the substrate concentration and the dilution rate using substrate concentration measurements corrupted by additive white noise $\upsilon(t)$. The sustained perturbation on the concentration of substrate flow feed was not included. The amplitude of the noise was taken as $|\upsilon(t)| \leq 0.1$. In this figure, the bandwidth of the noise on the substrate measurements was included. Note that the proposed controller maintains the substrate concentration in a small neighborhood of the reference value despite the noise on the substrate concentration measurements. As a matter of fact, the size of this neighborhood is smaller than the size of the noise bandwidth.

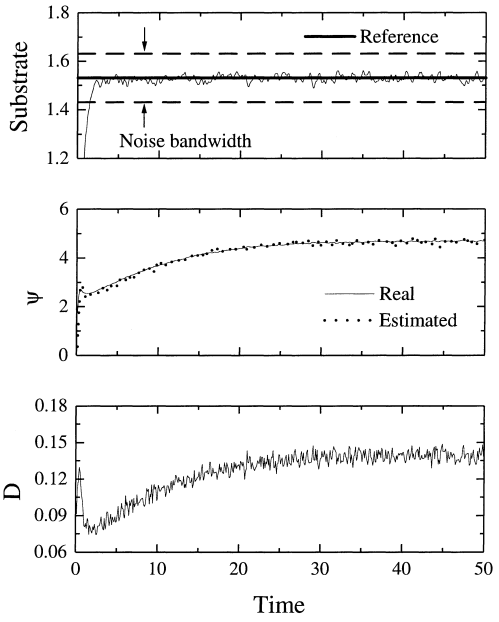

Fig. 5. Closed-loop performance with substrate concentration noisy measurements.

5. Future work

To obtain a better closed-loop performance it is necessary to eliminate the undesirable chattering effect. Some strategies have been proposed to face this problem. The most common approach [26] is to replace the sign function in (10) by a saturation function, $\mathrm{sat}(\tilde{x}_1/\phi)$, which is defined as

$$\mathrm{sat}\left(\frac{\tilde{x}_1}{\phi}\right) = \begin{cases} \dfrac{\tilde{x}_1}{\phi} & \text{if } \left|\dfrac{\tilde{x}_1}{\phi}\right| \leq 1 \\ \mathrm{sgn}\left(\dfrac{\tilde{x}_1}{\phi}\right) & \text{if } \left|\dfrac{\tilde{x}_1}{\phi}\right| > 1 \end{cases}$$

where ϕ is the boundary layer thickness which is made varying to take advantage of the system bandwidth. One drawback of varying the boundary layer is that for some systems the boundary width can become large. On the other hand, for some systems, the sat() function does not give satisfactory results [14]. Then, as future work, it is necessary to evaluate the closed-loop performance using different strategies for chattering reduction.

6. Conclusions

A control strategy to regulate a class of partially known bioreactors was proposed in this work. The proposed control algorithm takes advantage of the self-regulatory characteristic of the biological processes. The central idea of the proposed technique is to consider the modeling uncertainties as a new nonmeasurable state variable and to obtain estimates of this new state using an SMO. The conditions for practical convergence of the observer and the controller were developed. A characteristic of the resulting observer is that their parameters have physical meaning. The open-loop and closed-loop performance of the proposed controller was illustrated via numerical simulations. It was shown that this controller is able to regulate the substrate concentration despite modeling uncertainties, external sustained perturbations and noisy measurements.

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